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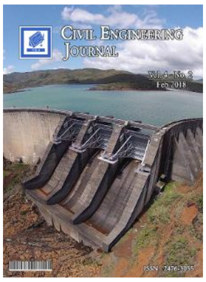

Damage Detection of Irregular Plates and Regular Dams by Wavelet Transform Combined Adoptive Neuro Fuzzy Inference System

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Abstract

This paper presents a technique for irregular plate and regular dam damage detection based on combination of wavelet with adoptive neuro fuzzy inference system (ANFIS). Many damage detection methods need response of structures (such as the displacements, stresses or mode shapes) before and after damage, but this method only requires response of structures after damage, otherwise many damage detection methods study regular plate but this method also studies irregular plate. First, the structure (irregular plate or regular dam) is modelled by using ANSYS software, the model is analysed and structure's responses with damage are obtained by finite element approach. Second, the responses at the finite element points with regular distances are obtained by using ANFIS. The damage zone is represented as the elements with reduced elasticity modules. Then these responses of structures are analysed with 2D wavelet transform. It is shown that matrix detail coefficients of 2D wavelet transform can specified the damage zone of plates and regular dams by perturbation in the damaged area.

Keywords: Damage Detection; Wavelet; Wavelet Transform; Fuzzy.

1. Introduction

In order to improve the sustainability of a building, it is necessary to pay more attention to the health of the structure, even in the buildings which have been normally designed for 100 years. Overloading, unauthorized uses, exposure to bad environmental conditions, and other unpredictable cases are among the factors affecting the demolition of a structure. Damage detection method has been an interesting topic in various fields for many researchers in recent years. Wavelet analysis is a mathematical method and a signal processing tool which is used with time frequency analysis to provide more information and details about the signals that cannot be analyzed by Fourier analysis. This method has various applications in many fields including civil, mechanical, and aerospace engineering, especially for structural damage Detection and structural health monitoring [1]. A simplified survey of governing theory on the wavelet transform and ability of wavelets to determine crack (damage) in the 2D continuum structures is presented in this paper, which only requires knowledge about response from the damaged structure.

Wang and Deng published a paper about damage detection using wavelet analysis [2]. Vestroni and Capecchi focused on the problem of locating and quantifying damage in vibrating beams due to cracks based on the minimization of the objective function that compares analytical and experimental data [3]. Kim and Melhem investigated a cracked beam specimen subjected to cyclic fatigue loads [1]. Yam et al proposed a method for damage detection of composite structures using combination of vibration responses, wavelet transform and artificial neural networks [4]. Lotfollahi-Yaghin and Koohdaragh suggested wavelet packet energy rate index to find out the characteristics of the crack [5]. Balafas, and Kiremidjian presented the development and validation of several novel data-driven damage sensitive

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features based on the Continuous Wavelet Transform [6]. Obrien et al. investigated a method for damage detection using a moving force identification algorithm [7]. Yu et al. studied damage detection in a six-bay truss bridge model and used the fuzzy C-means clustering algorithm to categorize features for structural damage detection [8]. El-Gebeily and Khulief developed methods for inner damage identification in pipes under noisy conditions for cracked beams with utilizing the vibration mode shapes [9]. Xu et al. proposed two–dimensional curvature mode shape method based on wavelets and Teager energy for damage detection in plates. The efficiency of the proposed method was demonstrated using finite element simulations and experimentally validated through noncontact measurement by a scanning laser vibrometer. The numerical results showed that its advances of clear mechanism of characterizing damage, robustness against noise, and sensitivity to slight damage were sufficiently corroborated [10]. Recently, He and Zhu have presented an adaptive–scale damage detection strategy based on a wavelet finite element model for thin plate structures. In this study, equations of motion and corresponding lifting schemes in thin plate structures have derived with the tensor products of cubic Hermite multi–wavelets as the elemental interpolation functions. The numerical results have demonstrated that the proposed method can progressively locate and quantify plate damages [11].

Khalil qatu et al. performed finite element simulation using ABAQUS/Explicit finite element software to study the damage size effect on the damage localization accuracy using the 3-point pulse-echo technique. A 5 cycle Gaussian tone-burst with 250 kHz central frequency is applied to simulate lamb wave propagation in an aluminium plate [12]. Solis et al. identified crack by using continuous wavelet analysis and difference between mode shapes. In this method, mode shapes of damaged and undamaged structures are needed. First, continuous wavelet transformation was applied to the mode shapes of damaged and undamaged structures so that their differences can be achieved in a damaged and undamaged state. Finally, the results of each mode shape were considered for the whole structure [13]. Montanari et al. introduced the effects of spatial sampling on crack detection in beams using continuous wavelet transform. They were looking at the following questions: (1) does the detection rate of cracking drop down with low sampling? 2) What is the optimal number of sampling intervals to identify the crack? First three modes shapes of the cantilever and simple support were evaluated. Finally, the optimal number of sampling intervals for crack identification was obtained [14]. Alamdari et al. used a two-dimensional discrete wavelet transform and proposed a method for identifying and determining the location of the crack by applying the frequency response functions of the damaged structure [15]. Chen and Oyadiji identified damage property from the modal frequency curve via discrete wavelet transform [16]. Ugo Andreaus et al. presented a method for crack detection and quantification in beams based on wavelet analysis. They simulated structures, using closed-form analysis for a given location of a concentrated load along the beam [17]. Hajizadeh et al. identified the damage type, damage existence and failure location in plate by wavelet and curvelet transform [18]. Obrien et al. investigated a method for damage detection using a moving force identification algorithm [19].

As stated in the previous paragraph, most research work on two-dimensional continuum structures damage detection using two-dimensional wavelet transform is related to regular structures, while the major structures that are available are irregular structures. In the case of three-dimensional structures, no significant research has been done. Both studies have been considered in this study and good results have been obtained. A simplified survey of governing theory on the wavelet transform and ability of combination of wavelet transform with ANFIS to determine damage in the irregular 2D continuum structures is presented in this paper, which only requires knowledge about response from the damaged structure. In other words, there is no need for information about the original undamaged (healthy) structure. In addition, by analyzing signal response from the static loads, the damage is locally specified.

2. An Overview of Wavelet Transform, ANFIS and ANSYS

2.1. Wavelet-Based Analysis

Wavelet analysis begins with the selection of a wavelet basic function among the available wavelets which are a function of location x or time t. This basic wavelet function is called mother wavelet, $\psi_{a,b}(x)$. Then, it would be delayed by a (stretched or pressed) and transferred by b in space to form a set of basic functions $\psi_{a,b}(x)$ [1].

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right) \tag{1}$$

The function is centralized in b with the spreading ratio of a. Wavelet transform (continuous or discrete) correlates wavelet function f(x) with $\boldsymbol{\psi_{a,b}}(x)$. Continuous wavelet transforms (CWT) decomposes a signal in the space domain into a two-dimensional function in the space-scale plane (a, b) as follows:

$$C(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(x)\psi\left(\frac{x-b}{a}\right) dx = \int_{-\infty}^{\infty} f(x)\psi_{ab}(x)\, dx \tag{2}$$

Where a, b are real numbers and a≠0. Results are wavelet coefficients which show how well a wavelet function is correlated with an analyzed signal. Therefore, sharp transforms produce wavelet coefficients with large magnitudes in f(x), which is also the basis for the proposed method.

Application of discrete scales can describe discrete wavelet transform (DWT) as follows:

$$C_{j,k} = 2^{-j/2}\int_{-\infty}^{\infty} f(x)\psi\left(2^{-j}x - k\right)dx = \int_{-\infty}^{\infty} f(x)\psi_{j,k}(x)dx \tag{3}$$

Resolution of signal is defined by the inverse scale, $\frac{1}{a} = 2^{-j}$, where the integer j is related to the level. The smaller the level and scale, the higher the resolution would be, and the more available the smaller components of the signal.

Note that the wavelet transform $C_{(a,b)}$ is possible for the small scales. $a < a_0$ And we want to recover f(x) function. In this case, we need complete information about $C_{(a,b)}$ for $a > a_0$. To collect this information, it is necessary to produce another function $\emptyset(x)$ which returns to the scale function (4). By replacing $\emptyset(x)$ with $\psi(x)$ in Equation 2, we get:

$$\mathrm{D(a,b)} = \int_{-\infty}^{\infty} \mathrm{f(x)}\frac{1}{\sqrt{\mathrm{a^0}}}\emptyset\left(\frac{\mathrm{x-b}}{\mathrm{a^0}}\right)\mathrm{dx} = \int_{-\infty}^{\infty} \mathrm{f(x)}\emptyset_{\mathrm{a_0,b}}\mathrm{(x)dx} \tag{4}$$

There is no scale function for each wavelet. Existence of function $\emptyset(\boldsymbol{x})$ for the numerical calculations of fast wavelet transform is very important. It must be noted that the dyadic scales are used for a, b and the reference level, J, must also be taken into account. By applying Equation (4) to this case, a set of coefficients will be obtained:

$$\mathrm{cD_J(k)} = \int_{-\infty}^{\infty} \mathrm{f(x)}\psi_{\mathrm{J,k}}\mathrm{(x)dx} \tag{5}$$

The coefficients $cD_J(k)$ are known as the level J detail coefficients. By applying the dyadic scale and level J, Equation (6) will tend to other set of coefficients.

$$\mathrm{cA_J(k)} = \int_{-\infty}^{\infty} \mathrm{f(x)}\emptyset_{\mathrm{J,k}}\mathrm{(x)dx} \tag{6}$$

The coefficients $\boldsymbol{cA_J(k)}$ are known as the approximation coefficients for level J. The function $\boldsymbol{D_j(x)}$is known as the detail function of level J.

$$\mathrm{D_j(x)} = \sum_{\mathrm{k=-\infty}}^{\infty} \mathrm{cD_j(k)}\psi_{\mathrm{j,k}}\mathrm{(x)} \tag{7}$$

$$\mathrm{A_J(x)} = \sum_{\mathrm{k=-\infty}}^{\infty} \mathrm{cA_j(k)}\emptyset_{\mathrm{j,k}} \tag{8}$$

The $A_J(x)$ is known as the approximation function at level J.

In damage detection, we are interested in the details of the signals. If f(*x*) is assumed to be a structural response (for example, a deformation curve), then $D_j(x)$ which including high frequency signal part has the necessary information for determining damage in the structure

2.1.1. Wavelets in Higher Dimensions

The wavelets in higher dimensions are obtained by the tensor product of one-dimensional wavelets. In short, this concept is expressed in a two-dimensional state, in which R^2 is considered with coordinate (x, y). Assume that $\emptyset$ and ψ are the scale function and wavelet for multi-resolution analysis (for example, Haar system or the system made by Daubechies). As shown in the previous section, the functions are [20]:

$$\phi_{jl}(x) = 2^{j/2}\phi\left(2^j x - l\right) \tag{9}$$

$$\psi_{jl}(x) = 2^{j/2}\psi\left(2^j x - l\right) \tag{10}$$

These bases are orthogonal. In addition, each ψ_{jl} is perpendicular to all ϕ_{jl}. For each set of indices $\acute{l}, \acute{j}, l$ and j functions:

$$\phi_{jl\acute{j}\acute{l}}(x,y) = \phi_{jl}(x)\phi_{\acute{j}\acute{l}}(y) \tag{11}$$

$$\psi_{jlj\acute{l}}(x,y) = \psi_{jl}(x)\psi_{j\acute{l}}(y) \tag{12}$$

Are defined where indices j and $\acute{j}$ vary between 0 and n (upper limit). The indices l and $\acute{l}$ correspond to the transferred components which depend on an arbitrary amplitude. For example, if the signal is defined on a unit square $\{(x,y): 0 \leq x, y \leq 1\}$, then $0 \leq l \leq 2^{j-1}$ and $0 \leq \acute{l} \leq 2^{j-1}$.

2.2. ANFIS

The fuzzy inference system, which uses adoptive training algorithms for neural networks, is called adoptive neuro fuzzy inference system, and is usually referred to as ANFIS. ANFIS is capable of constructing a fuzzy system whose parameters of its input and output membership functions are well-regulated using back-propagation neural network algorithm or a combination of the back-propagation algorithm and the least squares method.

The concept of ANFIS can be used as a useful way to solve function approximation problems. Figure 1, illustrates an example of the ANFIS structure, which consists of two inputs, four rules, and one output in the first-order fuzzy model of the Sugeno system. In this model, for convenience, it is assumed that there are two membership functions for each input. These assumptions are intended to simplify the understanding of the ANFIS structure, otherwise it is clear that this structure can be generalized to any arbitrary dimension [21].

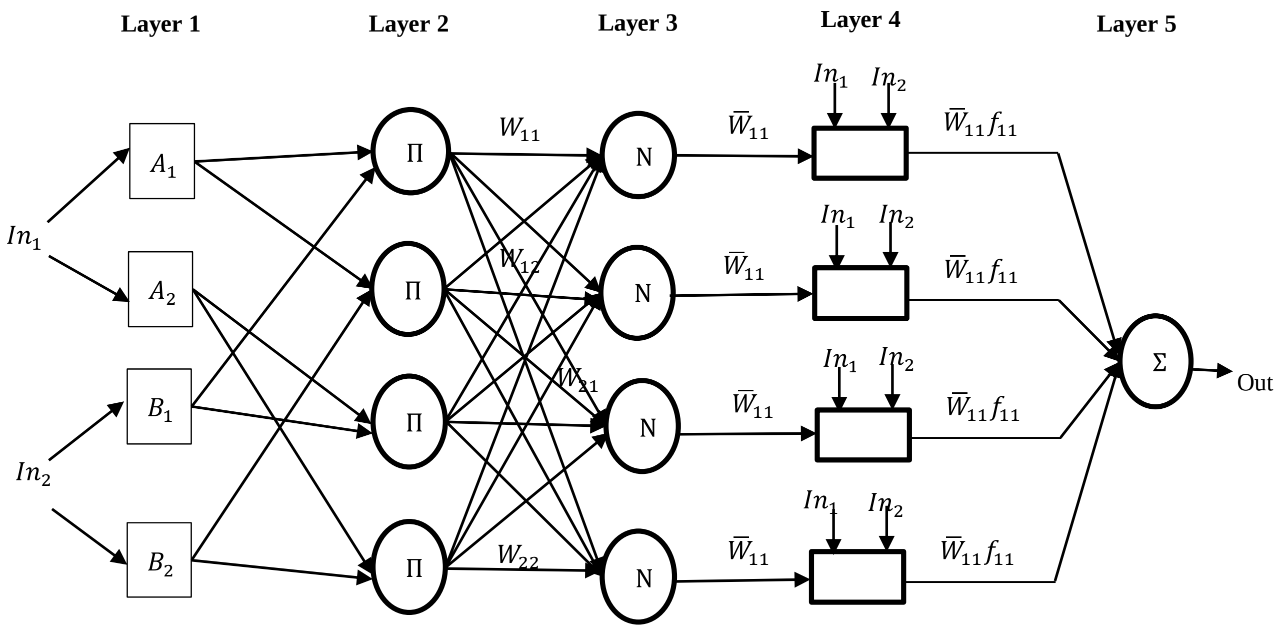

Figure 1. ANFIS sample structure in Sugeno fuzzy model

Which A_i, B_j $(i,j = 1,2)$ are symbols for the membership functions of the two inputs In_1, In_2 respectively and $W_{ij},\ \bar{W}_{ij}$ are weight factor.

2.3. ANSYS

ANSYS structural analysis is a software which capable to solve complex structural engineering. With the finite element analysis (FEA) tools can automate solutions for structural mechanics problems and parameterize them to analyze multiple design scenarios. ANSYS can connect easily to other software. ANSYS structural analysis software is used to enable engineers to analyze, design and optimize structures. In this paper ANSYS is used to obtain structural responses of two irregular plates and a regular dam.

3. Damage Detection

3.1. Image Processing

Before discussing the damage detection in 2D structures, we first explain how the images are processed. Consider the following image Figure 2, this image is divided into regular networks with equal intervals. Based on the colors of the points, intensity of the color is also determined which is usually between 0 and 255.

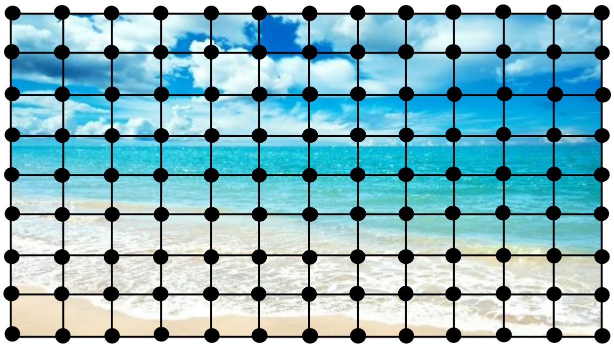

Figure 2. This image is divided into regular networks with equal intervals

By this method, the image is changed to a 2D matrix whose elements represent the light intensity around the corresponding point on the image. The smaller the distance between the points on the image, the more the resolution and accuracy of the image and the larger the dimension of the corresponding matrix would be. It must be emphasized that these points should be selected with equal intervals on the image. Now, by applying the wavelet transform to this 2D matrix, four matrices would be obtained where one is the approximate matrix and the other three are details matrices. If dimension of the main matrix for the image is $2m \times 2n$, dimension of all the resulting matrices will be $m \times n$. Consider the following diagram (Figure 3):

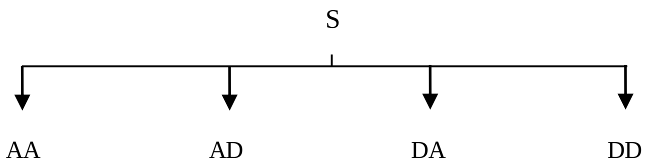

Figure 3. Applying the 2D discrete wavelet transform to the first level of 2D matrix.

If S is the original matrix, at the first level, AA is the approximate matrix of columns and rows of matrix S, AD is the approximate matrix of row and column details of matrix S, DA is the details matrix of row and column approximate of matrix S, and DD is details matrix of row and column of matrix S.

3.2. Damage Detection in Regular Two-Dimensional Structures

The following plate Figure 4 is a geometrically-regular two-dimensional structure, on which an arbitrary loading effect was applied. The structure was supposed to have a fault in the hachured area.

Inspired by the above-mentioned image processing method, the structure was modeled by finite elements with the same size using ANSYS software and, then, the model was analyzed. The desirable structural response (displacement, stress, etc.) was obtained at some points of the structure with equal distances (under experimental conditions, the sensors are located at these points). In other words, instead of light intensity in image processing, the magnitude of the response (displacement, stress, etc.) was obtained at some points of the structure with equal distance to each other Figure 5.

By arranging these responses next to each other, a two-dimensional matrix was obtained such that dimension of the matrix was exactly the same as that of finite element points on the structure. By applying the wavelet transform to this two-dimensional matrix, DD matrix (matrix representing details in the column and row) was obtained. Then, by examining this matrix, a jump was observed at the fault point.

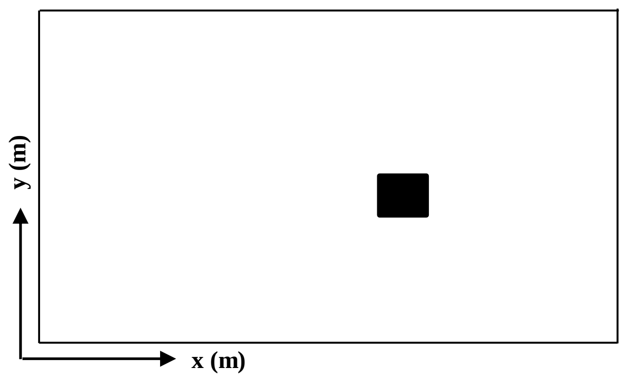

Figure 4. Geometrically-regular two-dimensional plate with a fault

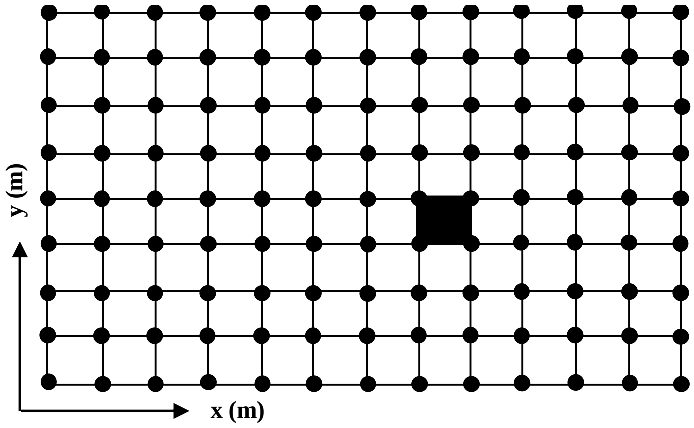

Figure 5. The plate was modeled by finite elements with the same size using ANSYS software

3.3. Damage Detection in Irregular Two-dimensional Structures

The presented plate Figure 6 is a geometrically-irregular two-dimensional plate. When the plate was modeled in the analyzing software, its elements were configured according to the following Figure 7.

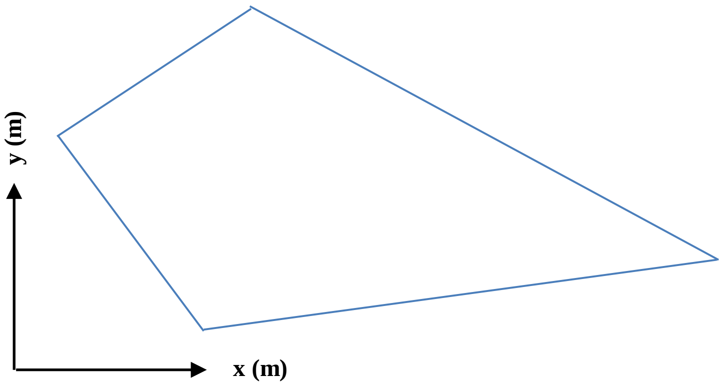

Figure 6. Geometrically-irregular two-dimensional plate

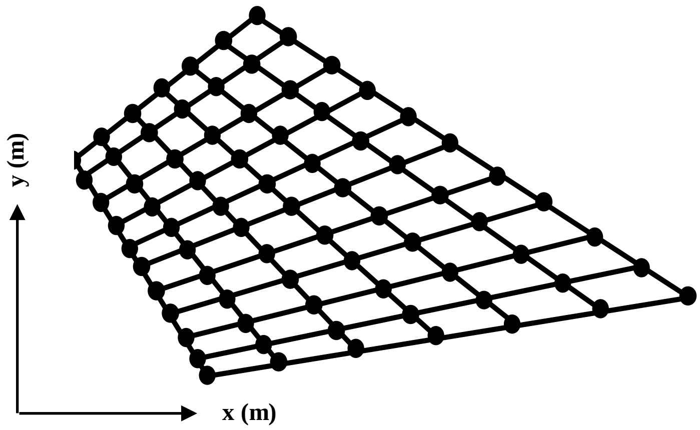

Figure 7. The plate was modeled in the analyzing software

The response (displacement, stress, etc.) was obtained at finite element points; however, in this case, the points were not equally spaced in the vertical and horizontal directions any more (under experimental conditions, it is impossible to install the sensors within equal distance to each other), while the structural response at the points with equal and regular distances was needed.

To solve this problem, the considered plate was supposed to be located inside a regular plate Figure 8. If this regular plate were divided into regular vertical and horizontal distances, the following mesh Figure 9 would be obtained.

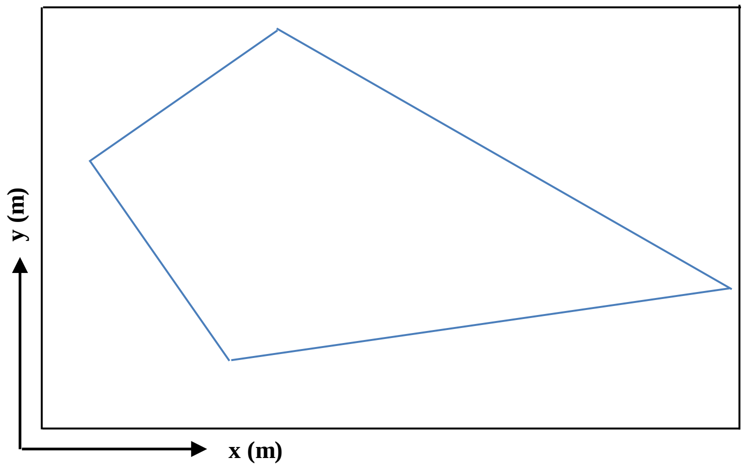

Figure 8. The irregular plate was supposed to be located inside a regular plate

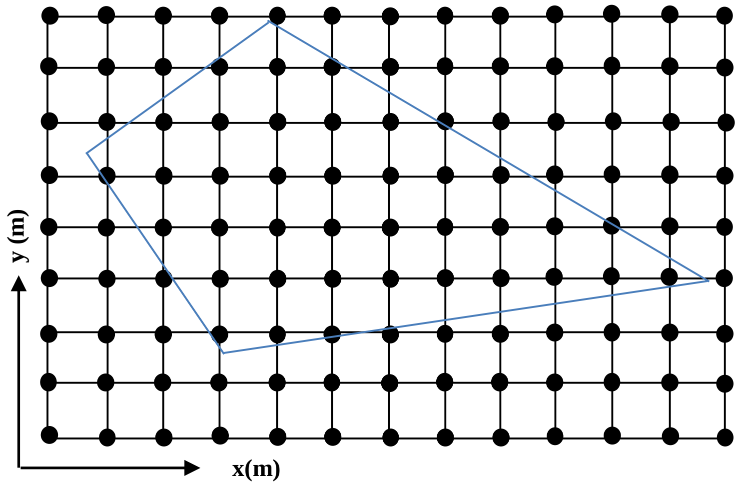

Figure 9. The mesh would provide the expected regular two-dimensional matrix

This mesh would provide the expected regular two-dimensional matrix, but with two problems: 1-There were some points in this meshing that are located outside the studied structure, to eliminate this problem, responses for these points were assumed to be zero, since they did not actually exist. 2-There were some points on the studied structure; but, they were not the points whose response was obtained from finite element analysis. There are two methods for obtaining the structural response at these points.

First, according to the coordinate of the considered points, the finite elements within which coordinates of these points are located are analyzed. And, the magnitude of the response at the considered points is obtained based on the governing equations. This method is highly time-consuming and practically impossible due to the high number of these points. The second method is to use ANFIS to obtain the response at points with regular distances to the response of finite element points with irregular distances. In order for the ANFIS to have better performance, it is recommended to add some points near or outside the structure's boundary with zero value to the mesh input points. The correlation coefficient is used to estimate accuracy of ANFIS. The program is repeated to achieve high accuracy. The mathematical formula for computing correlation coefficient between the two variables (x, y) is

$$f(x,y)=\left(\frac{n\sum xy-(\sum x)(\sum y)}{\sqrt{n(\sum x^2)-(\sum x)^2}\sqrt{n(\sum y^2)-(\sum y)^2}}\right) \tag{13}$$

Where, f is correlation coefficient and n, is number of pairs of data, x is real data, y is approximation data. If x and y have a strong positive linear correlation, correlation coefficient is close to number +1. A correlation coefficient value of exactly +1 indicates a perfect positive fit.

After assuring the high accuracy of the calculations, by ANFIS, it is time to calculate the structure response by ANFIS at points with equal intervals. By summing up these responses, the two-dimensional structural response matrix is formed. By applying a two-dimensional discrete wavelet transform to this matrix, the DD matrix is obtained.

3.4. Damage Detection in Regular Dams

The presented dam Figure 10 is a regular dam, on which an arbitrary loading was applied. When the dam was modeled in the analyzing software, its elements were configured according to the following mesh. The dam can be cut by sheets in the direction of one dimension where each cut would be considered as an irregular plate Therefore, the process that were used for irregular plate can be used for each sheet of the dam.

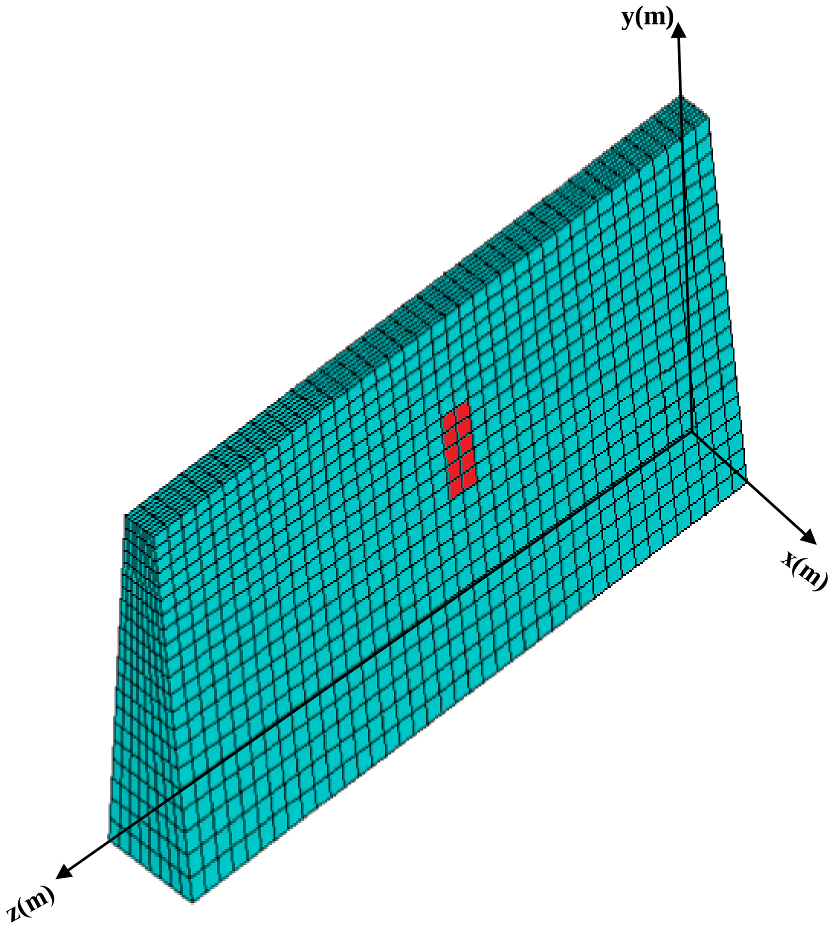

Figure 10. irregular dam with one damage zone

3.5. Flowchart of Proposed Method

For a better explanation of the proposed method, the flowchart of the various steps of the method is shown in Figure 11.

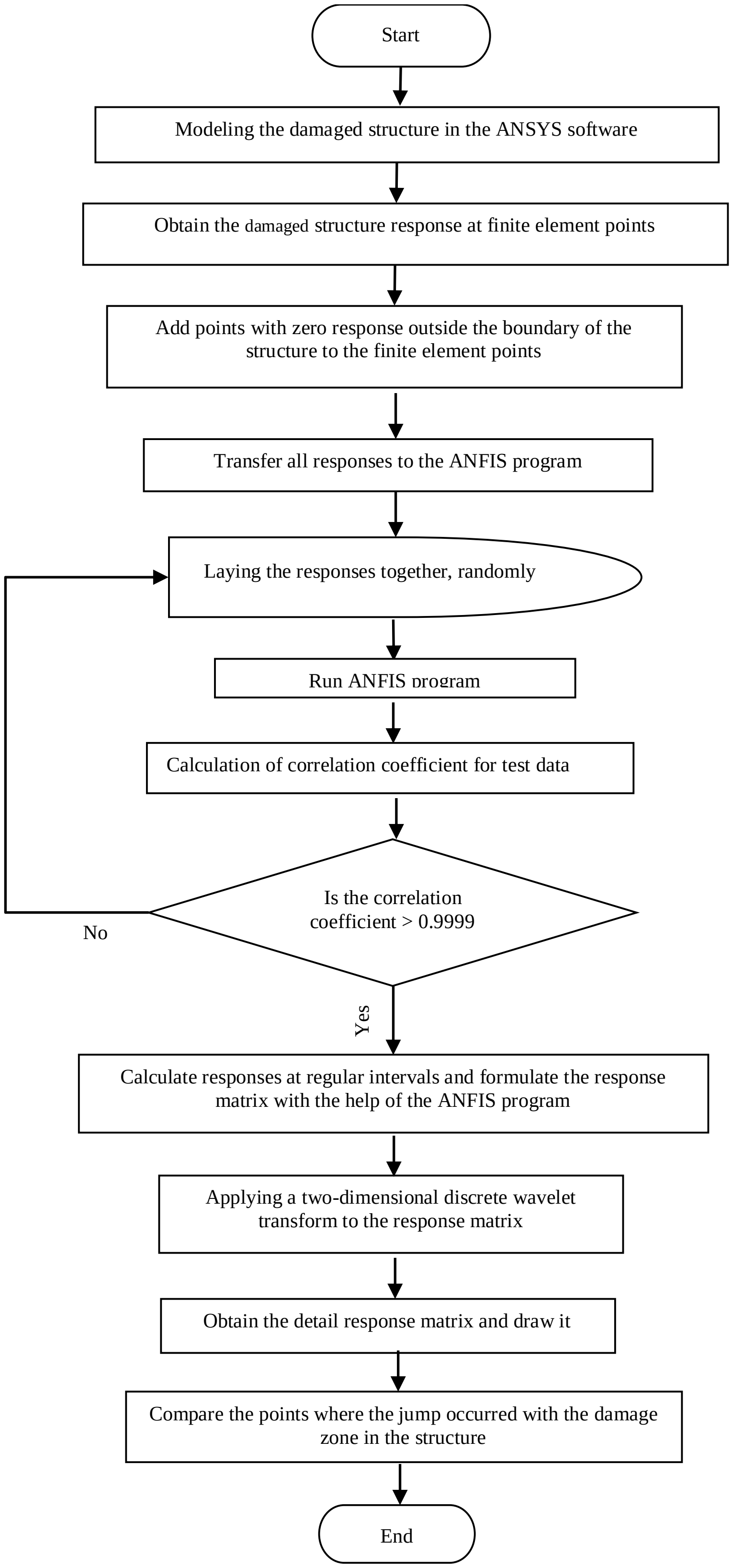

Figure11. The method flowchart

4. Numerical Examples

In order to show the capabilities of the proposed method for identifying structural damage, four illustrative test examples are considered. The first example is an irregular plate with one damaged zone, the second one is a plate similar to the previous one, but with four damaged zones, the third example is an irregular plate with two damaged zone and the fourth one is a regular dam.

4.1. Plate with One Damaged Zone

The presented plate shown in Figure 12 is considered with the height of 4m, width of 7 m at the top, thickness of 0.10m, and elasticity module of $1.5\,E9\ ton/m^2$. In the darker area shown by number (2), elasticity module of the plate was reduced by 50%. It has fixed support in top, bottom, left and right border and a hydro static load ($4000\,Kg/m^2$).

The mentioned plate was modeled, loaded, and analyzed by finite elements method. The displacement was considered as the response. The responses were provided as the input to the ANFIS program. Conformity of outputs data and targets data which formed in ANFIS program are shown in Figures 13 and 14.

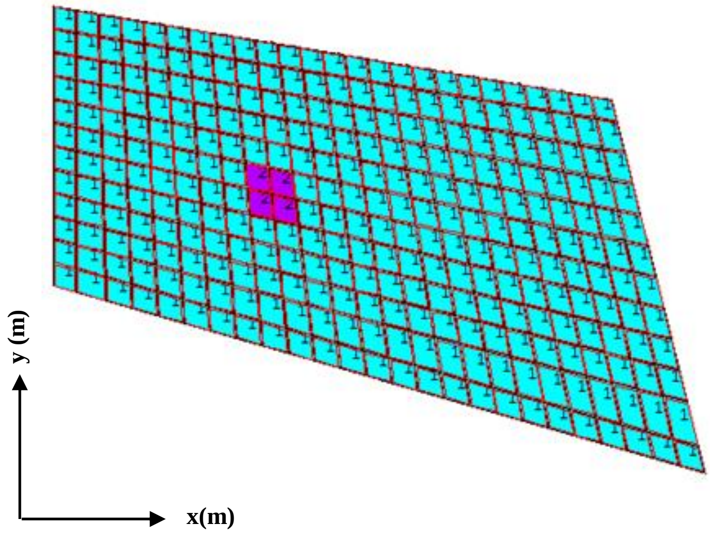

Figure 12. plate with one damage zone

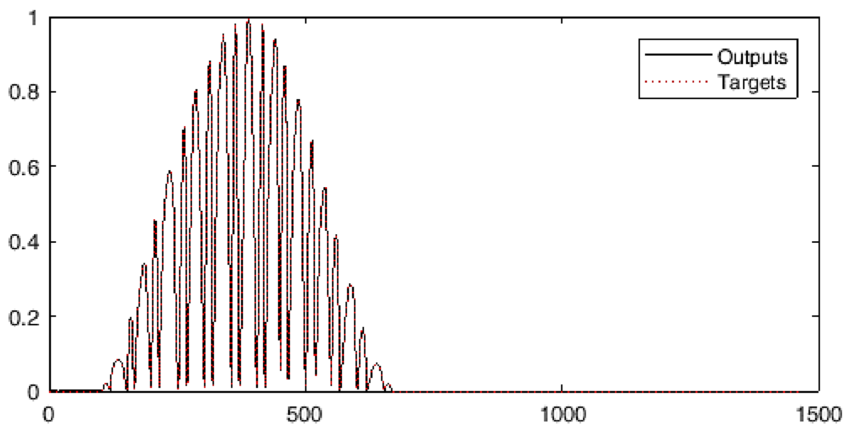

Figure 13. Conformity of outputs data and targets data

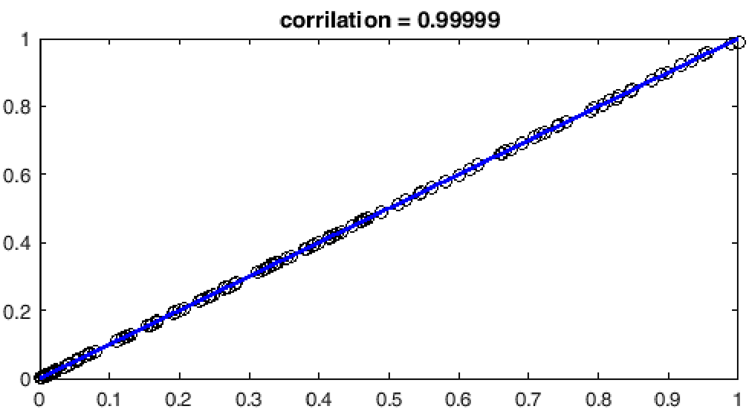

Figure 14. Correlation coefficient

In the next step, the original 2D matrix representing plate response was formed by ANFIS program. This matrix was also analyzed using the 2D wavelet transform (mother wavelet is shown by Equation 12 for the displacement responses, and in the DD matrix, a jump in the damaged area is shown in Figure 15. In this figure, the x and y axes indicate the half number of points (HNP) that the ANFIS calculates the response matrix at those points and in those directions.

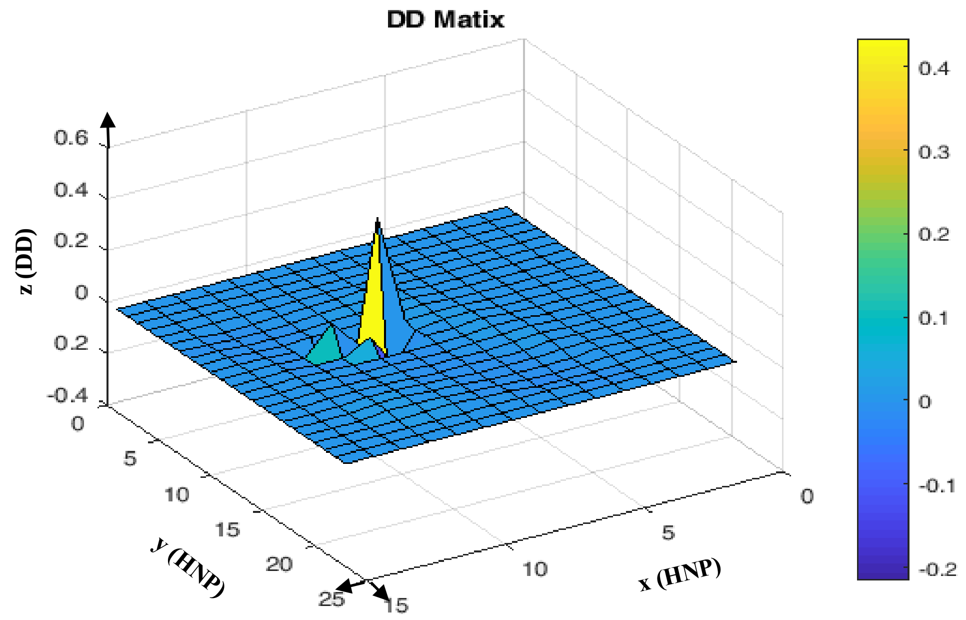

Figure 15. DD Matrix

4.2. Plate with Four Damaged Zones

The presented plate shown in Figure 16 is identical to the previous plate with the exception of having four damaged zones. In the regions shown with number (2), elasticity module of the plate was reduced by 40% and in the regions shown with number (3), elasticity module of the plate was reduced by 70%.

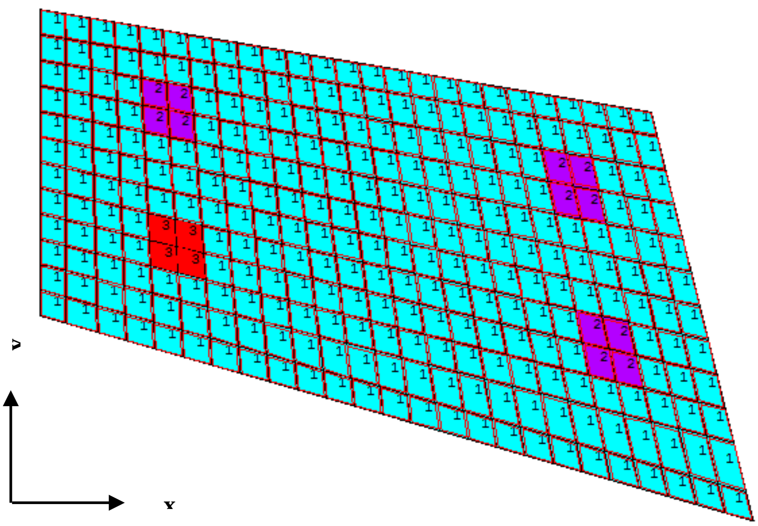

Figure 16. Plate with four damaged zones

The above-mentioned plate was modeled, loaded, and analyzed by a finite elements method, the principal stress (S1) was considered as the response. The DD matrix was obtained by applying 2D wavelet transform to the original 2D matrix which was formed by using the ANFIS program. It is shown that matrix DD can specify the damaged zone of plate by perturbation in these areas Figure 17.

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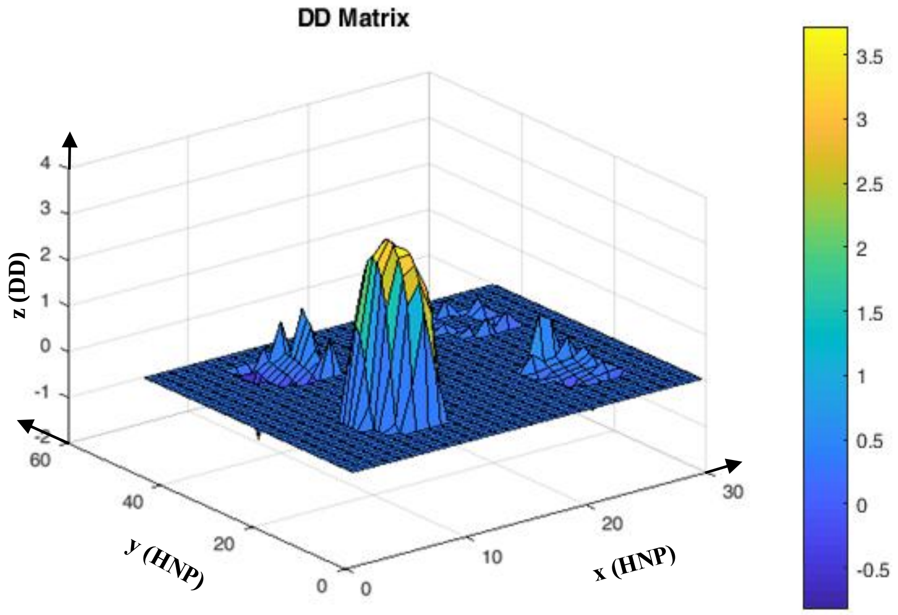

Figure 17. DD Matrix

4.3 Plate with Two Damaged Zone

The presented plate shown in Figure. 18 is considered with the height of 8m, width of 4m at the top, width of6m at the bottom, thickness of 0.10m, and elasticity module of $1.5\,E9\ ton/m^2$. In the area shown by number (2), elasticity module of the plate was reduced by 50%. It has fixed support in left and right border and a uniform load ($2000\,Kg/m^2$).

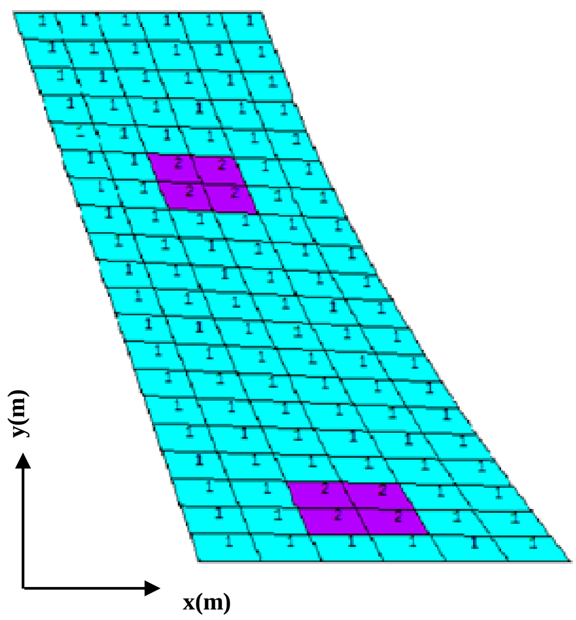

Figure 18. Plate with two damage zones

The above-mentioned plate was modeled, loaded, and analyzed by a finite elements method, the principal stress (S1) was considered as the response. The DD matrix was obtained by applying 2D wavelet transform to the original 2D matrix which was formed by using the ANFIS program. It is shown that matrix DD can specify the damaged zone of plate by perturbation in these areas Figure 19.

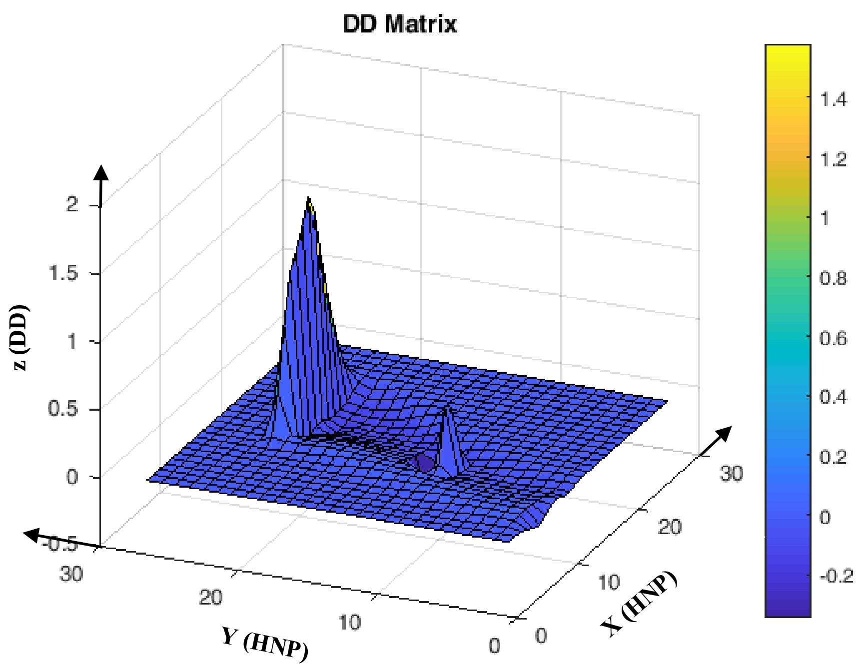

Figure 19. DD Matrix

4.4. Dam with One Damaged Zone

The presented dam shown in Figure 10 is considered with the height of 40m, width of 5m at the top, width of 15m at the bottom and length of 100m, and elasticity module of $1.5\ E9\ ton/m^2$. In the darker zone, elasticity module of the dam was reduced by 50%. It has fixed support in bottom, left and right border and a hydro static load. It is shown that matrix DD can specify the damaged zone of dam by perturbation in these sections Figure 20 and can specify the undamaged zone of dam without perturbation in these sections Figure 21.

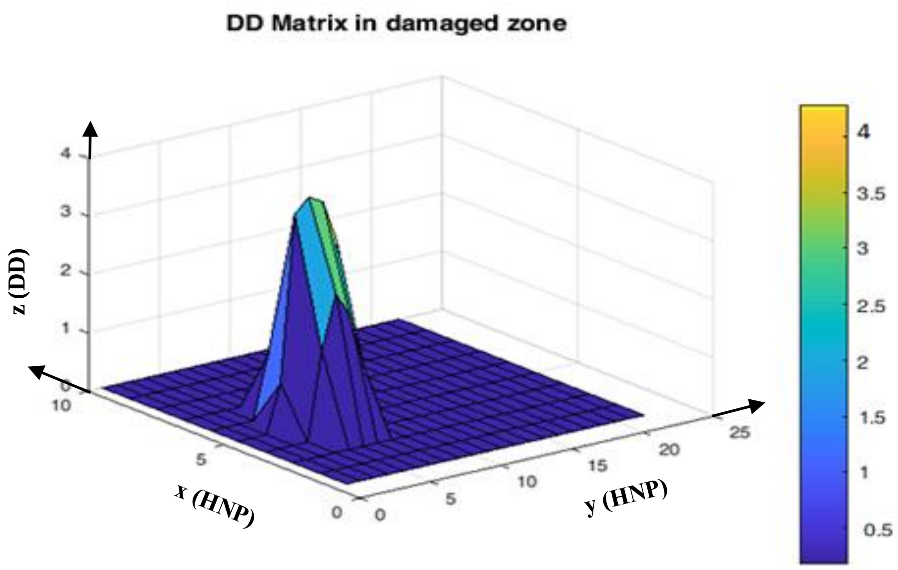

Figure 20. DD matrix in the damaged section of dam

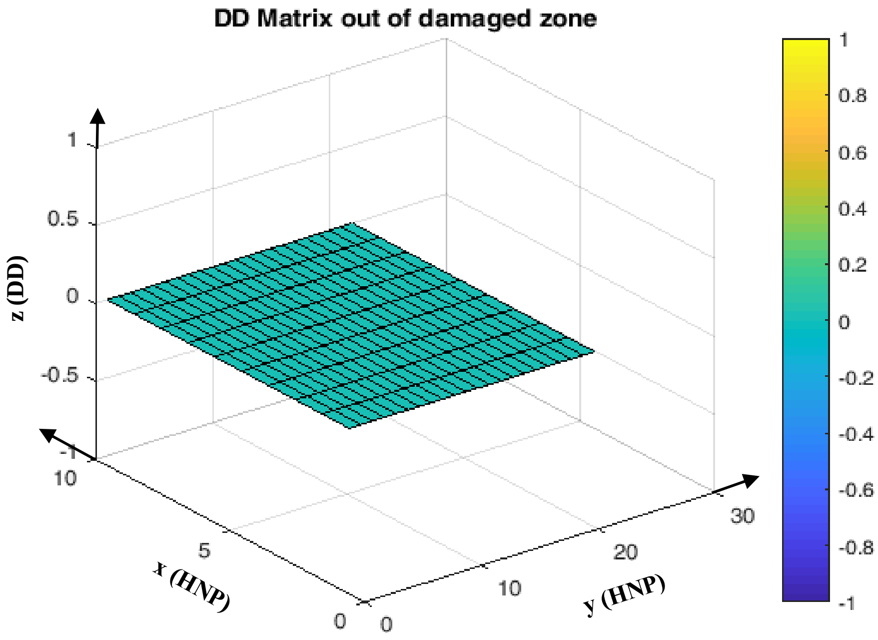

Figure 21. DD matrix in the undamaged section of dam

5. Conclusion

A new approach is proposed for irregular structure damage detection based on combination of wavelet with ANFIS. The plate was modeled, loaded, and analyzed. The plate response (displacement, stress) was obtained at finite element points; however, in this case, the points were not equally spaced in the X, Y directions any more (under experimental conditions, it is impossible to install the sensors within equal distance to each other), while for 2D wavelet transform, the structural response at the points with equal and regular distances was needed. To solve this problem, ANFIS is used to obtain the response at the finite element points with regular distances to the response of finite element points with irregular distances. The main conclusions are as follows:

- The ANFIS method which is used to estimate the structural responses of the irregular points to a regular domain is suitable for wavelet transform.
- Structural responses matrix of irregular plate can be obtained by transferring the irregular plate inside a regular imaginary plate.
- By adding a number of points near or outside the structure's boundary with zero value to the mesh input points more accurate results are achieved in ANFIS.
- The details matrix which is obtained by applying 2D wavelet transform to plate response matrix can specify the damaged zones of plate by perturbation in these areas.
- The process that were used for irregular plates can be used for regular dams.

6. References

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